Online Fair Division with Unequal Entitlements

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In 2014, Foodbank Australia distributed more than 100,000 meals per day, but could not assist around 60,000 people: 40% were children. The foodbank struggles for a better allocation mechanism. We therefore formulate an *online fair division* model for their problem in which food arrives online and a mechanism allocates it to charities *with unequal entitlements*.

Model and Mechanisms

Suppose k agents. Item j arrives at time j. Each agent a_i has *utility* u_i for and *entitlement* e_i to this new item. They bid for item j and a mechanism allocates it according to the bids and the entitlements of the agents for it. We consider two simple mechanisms. Each mechanism computes a set of feasible agents for item j and allocates it at random to a feasible agent with a probability proportional to their entitlement with respect to the other feasible agents.

- the Entitlement mechanism: agent a_i is feasible if they bid positively
- the Fair Entitlement mechanism: agent a_i is feasible if they bid positively and have fewest items relative to their entitlement

Properties

We present 5 axiomatic properties of Entitlement and Fair Entitlement that are practically important.

Example: Consider the online fair division of items 1, 2 and 3 amongst agents a_1 (green line), a_2 (red line) and a_3 (black line). Let $a_1/a_2/a_3$ have utility of 1 for all items/item 2/items 1 and 3. Each other utility is equal to 0.

- efficiency: Both Entitlement and Fair Entitlement are efficient with 0/1 utilities.
- monotonicity: Entitlement is monotonic (i.e. increasing your entitlement increases your allocation), but Fair Entitlement is not.

Figure 1: Agents a_1, a_2 and a_3's entitlements are equal to (Left) $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$ and (Right) $\frac{3}{4}$, $\frac{1}{4}$ and 0.

  e.g. agent a_1 receives expected utility of $\frac{11}{9}$ if e_1 equals $\frac{1}{2}$ and **lower** expected utility of 1 if e_1 **increases** to $\frac{3}{4}$
- envy-freeness: Fair Entitlement is bounded envy-free ex post, but Entitlement is not.

  e.g. agent a_1 may get all items in which case a_3 envies a_1 by assigning 2 units of utility greater to their allocation than to their own allocation
- strategyproofness: Entitlement is strategyproof, but Fair Entitlement is not.

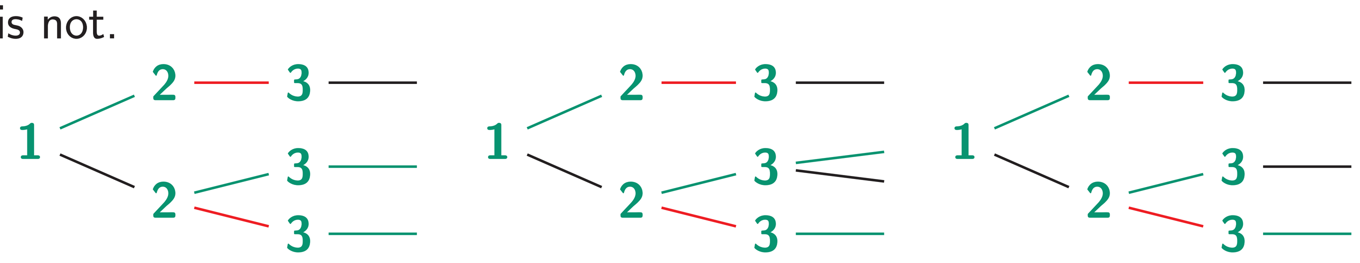

Figure 2: Agents a_1, a_2 and a_3's entitlements are equal to (Left) $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$, (Center) $\frac{1}{3}$ and (Right) $\frac{1}{4}$, $\frac{1}{4}$ and $\frac{1}{2}$.

  e.g. agent a_1 receives expected utility of $\frac{11}{9}/\frac{9}{8}/1$ (Left/Center/Right) if they bid **sincerely** 1 for item 1 and **greater** expected utility of $\frac{5}{3}/\frac{5}{4}$ (Left/Center) or the **same** expected utility of 1 (Right) if they bid **strategically** 0 for it
- egalitarianism: We measure the (1) ratio between the egalitarian and optimal welfares (i.e. *Egalitarian ratio*), (2) half of the relative mean absolute difference (i.e. *Gini index*) and (3) utility amount we need to get from the "rich" agents and redistribute among the "poor" agents in order to achieve uniformity of the expected utilities relative to the entitlements (i.e. *Robin Hood index*).

Group Behavior

A group of agents are from the same *type* whenever, for each item, their bids are either all positive or equal to zero.

- type anonymity: given the bids of the agents, each agent from a type gets each item with the same probability
- typeproofness: given the utilities of the agents, each agent belongs to the same type across all equilibria

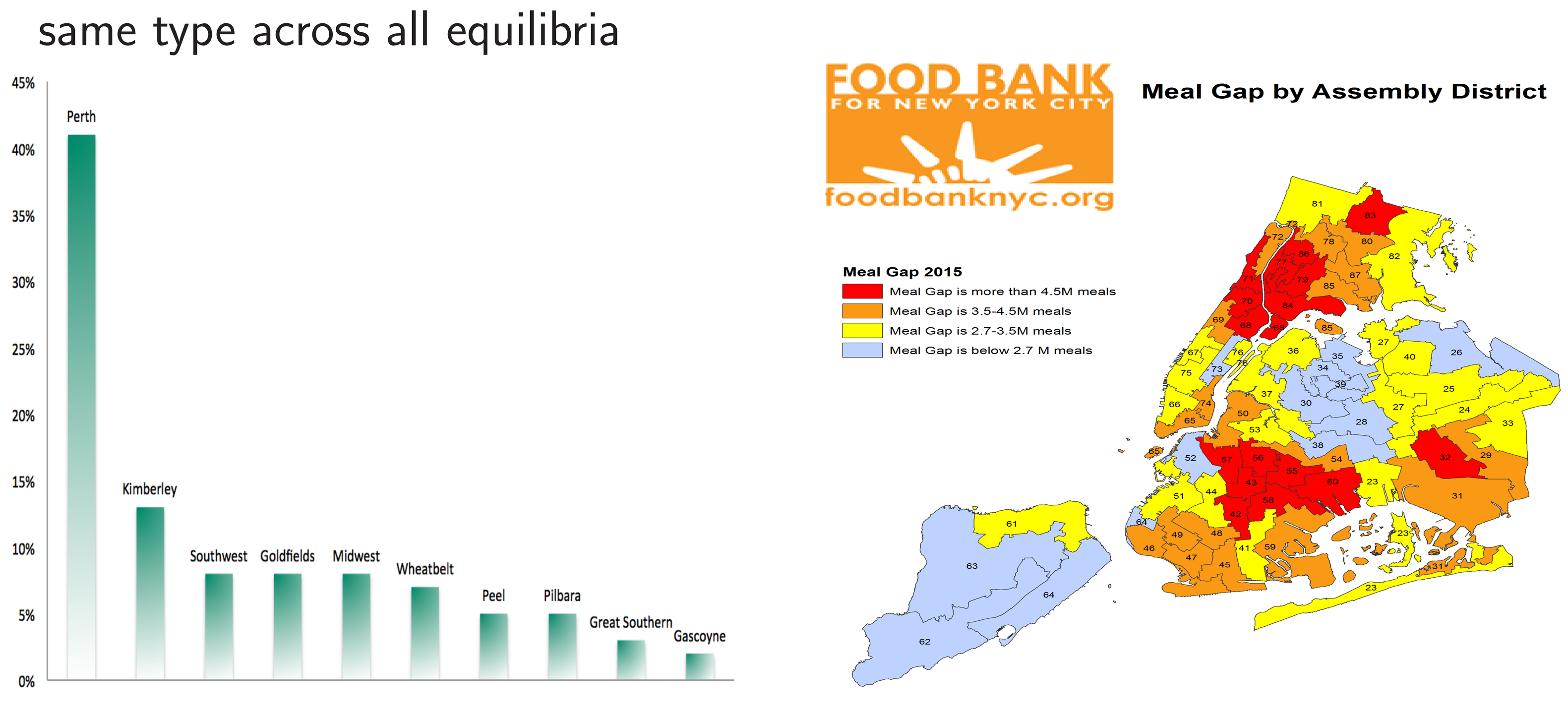

Figure 3: (Left) District Entitlements in Western Australia in 2014. (Right) The Meal Gap in New York City in 2015.

Experiment

Problem: Nearly 24,000 children with no meal from Foodbank Australia.
Goal: School Breakfast Program aims to cover this gap by donating nearly 8700 breakfasts daily to more than 300 schools in Western Australia.
Setting: 10 agents (one per district), 100 items, 0/1 utilities
Result: Fair Entitlement achieves (1) nearly 35 times smaller envy-freeness bounds and (2) around 10% increased welfare compared to Entitlement.

Mechanism	Entitlement		Fair Entitlement	
Entitlements	**equal**	**unequal**	**equal**	**unequal**
Bound of envy-freeness ex post	32.52	507.12	0.81	14.17
Egalitarian ratio (r)	0.83	0.72	0.94	0.85
Gini index (g)	0.06	0.10	0.04	0.07
Robin Hood index (h)	0.04	0.07	0.02	0.05

Table 1: Overview of results over 100 instances: r's optimal value is 1, g and h's optimal values are 0.

Summary & Future Work

Mechanism	Entitlement		Fair Entitlement	
Utilities	**binary**	**general**	**binary**	**general**
Efficiency ex post/ex ante	✓,✓	$\times^2,\times^2$	✓,✓	$\times^2,\times^2$
Utility/Entitlement monotonic	✓,✓	✓,✓	$\times^3,\times^3$	$\times^2,\times^2$
Agent/Item monotonic	✓,✓	✓,✓	$\times^3,\times^3$	$\times^2,\times^2$
Envy-free ex post/ex ante	$\times^2$,✓	$\times^2$,✓	$\times^2$,✓	$\times^2,\times^2$
Bound of envy-freeness ex post	∞^2	∞^2	$\lceil\frac{E}{e}\rceil$	∞^2
Anonymous/Neutral	✓,✓	✓,✓	✓,$\times^2$	✓,$\times^2$
Strategyproof	✓	✓	$\times^3$	$\times^2$
Type anonymous	✓	✓	✓	✓
Equal treatment of equals	✓	✓	✓	✓
Typeproof	✓	✓	✓	$\times^2$
Strategyproof with types	✓	✓	$\times^3$	$\times^2$
Egalitarian/Utilitarian ratio	$\frac{1}{e}$,1	$\frac{1}{e}$,n	$\geq\frac{1}{e}$,1	∞^2,∞^2
Egalitarian/Utilitarian price	$\frac{1}{e}$,1	$\frac{1}{e}$,n	$\geq\frac{1}{e}$,1	$\geq\frac{1}{e},\geq n$

Table 2: Overview of properties for sincere play: k agents, n items, minimum e and maximum E entitlements.

- At any round, multiple items are expected to arrive and we can apply offline mechanisms for allocating these items.
- We further investigate various agent and authority control manipulations from complexity point of view and are in search for tractable islands.

FOR FURTHER INFORMATION

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