Online Mechanisms for Sustainable Fair Division

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Online Fair Division provides a framework that reflects the manner in which we allocate resources over time to agents supposing only partial information about their preferences is available. The online decisions made today could improve the allocation tomorrow. For this reason, an ideal mechanism for online fair division must support the sustainability of the fair divisions by allocating the future resources *efficiently* and *fairly*.

Online Model 1: Donated Food Allocations

Suppose there are $\mathbf{n}$ *agents* and $\mathbf{m}$ *items*. Each agent $\mathbf{a_i}$ has some *utility* $\mathbf{u_{ij}}$ for each item $\mathbf{j}$. At time moment $\mathbf{j}$, (1) item $\mathbf{j}$ arrives, (2) each agent $\mathbf{a_i}$ *bids* for $\mathbf{j}$ and (3) a *mechanism* allocates this item to an agent.

Online randomized allocation mechanism computes, at round $\mathbf{j}$, a set of *feasible* agents for item $\mathbf{j}$ and assigns it uniformly at random to a feasible agent.

- LIKE mechanism: agent $\mathbf{a_i}$ is feasible for item $\mathbf{j}$ if they bid positively for it
- BALANCED LIKE mechanism: agent $\mathbf{a_i}$ is feasible for item $\mathbf{j}$ if they bid positively for it and have fewest items among those bidding positively for it

Suppose p_{ij} is the probability of agent $\mathbf{a_i}$ for item $\mathbf{j}$. The **expected utility** of agent $\mathbf{a_i}$ over all $\mathbf{m}$ items can be given as

$$\overline{u}_i = \sum_{j=1}^{m} p_{ij} \cdot u_{ij}. \tag{1}$$

Axiomatic Properties

We present four axiomatic properties of LIKE and BALANCED LIKE that are practically important and commonly studied in theory.

Example: Let us consider agents $\mathbf{a_1}$ (green edges), $\mathbf{a_2}$ (red edges) and $\mathbf{a_3}$ (black edges) and items $\mathbf{1}$, $\mathbf{2}$ and $\mathbf{3}$. Suppose the cardinal utilities for $\mathbf{1}$, $\mathbf{2}$ and $\mathbf{3}$ are as follows: $\mathbf{a_1}$ has 1, 1, 1, $\mathbf{a_2}$ has 0, 1, 0 and $\mathbf{a_3}$ has 1, 0, 1.

- efficiency: With our mechanisms, the allocation is ex post efficient.
- envy-freeness: With LIKE, $\mathbf{a_1}$ might get all items when $\mathbf{a_3}$ envies $\mathbf{a_1}$ with 2.
- strategyproofness: LIKE is strategyproof whereas BALANCED LIKE is not.

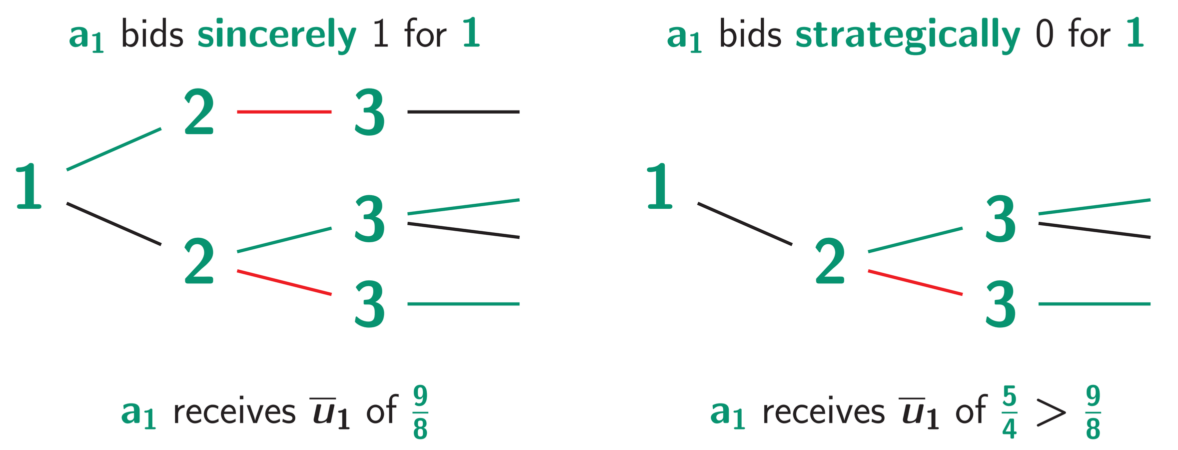

Figure 1: Strategic play for BALANCED LIKE.

- competitiveness: The offline egalitarian ratio is the ratio between the welfare, i.e. $\mathbf{min}_i\, \overline{u}_i$, obtained with an online mechanism and the welfare returned by the optimal (offline) mechanism.

Property	LIKE		BALANCED LIKE	
	binary	**general**	**binary**	**general**
Efficiency ex post/ex ante	$\checkmark,\checkmark$	$\times^2,\times^2$	$\checkmark,\checkmark$	$\times^2,\times^2$
Envy-freeness ex post/ex ante	$\times^2,\checkmark$	$\times^2,\checkmark$	$\times^2,\checkmark$	$\times^2,\times^2$
Bound of envy-freeness	$\mathbf{m}$	$\mathbf{m \cdot u}$	$\mathbf{1}$	$\mathbf{m \cdot u}$
Strategyproofness	$\checkmark$	$\checkmark$	$\times^3$	$\times^2$
Offline egalitarian ratio	$\mathbf{n}$	$\mathbf{n}$	$\geq \mathbf{n}$	∞^2

Table 1: Overview of properties for sincere play (see [1]): $\mathbf{n}$ agents, $\mathbf{m}$ items, maximum utility $\mathbf{u}$ and constant $\mathbf{e}$.

Online Model 2: Deceased Organ Matchings

Suppose there are $\mathbf{k}$ *patients* on a strict waiting list and $\mathbf{l}$ deceased *organs*. Each patient $\mathbf{p_i}$ has two indicators: *blood type* (e.g. O, AB) and *estimated post-transplant survival* index v^i_{EPTS}. Each kidney $\mathbf{j}$ has two indicators as well: blood type and *kidney donor profile* index v^j_{KDPI}. At time moment $\mathbf{j}$, (1) pair of organs $\mathbf{j_1}$ and $\mathbf{j_2}$ are donated and (2) a *mechanism* matches these organs to patients. Each patient $\mathbf{p_i}$ has *utility* $-|v^i_{EPTS} - v^j_{KDPI}|$ for organ $\mathbf{j}$. At the next moment, some agents may arrive at or depart from the waiting list.

Constraints:

- each patient **must** be matched to organs of compatible blood type, e.g. patient of type O can only be matched to organs of type O
- each patient **prefers** to be matched to organs that maximize their utility

Online deterministic matching mechanism computes, at round $\mathbf{j}$, a set of *feasible* patients for organ $\mathbf{j}$ and matches it to the patient who waited most.

- HARDTYPEBESTINDEX mechanism: patient $\mathbf{p_i}$ is feasible for organ $\mathbf{j}$ if their blood types are exact and minimizes $|v^i_{EPTS} - v^j_{KDPI}|$
- SOFTTYPEBESTINDEX mechanism: patient $\mathbf{p_i}$ is feasible for organ $\mathbf{j}$ if their blood types are compatible and minimizes $|v^i_{EPTS} - v^j_{KDPI}|$

Note: "Blood type" ties are broken in favor of more compatible type (see [2]).

Axiomatic Properties

The model is novel because the organs are matched online and the matching decisions made by the mechanisms are based on *multiple* criteria: blood type and quality index. To capture their comlexity, we present three generalized properties.

- quality efficiency: Each patient is matched to an organ of most compatible quality index and any other organ match for them is at most as good as this one with respect to their index.
- quality envy-freeness: Patient $\mathbf{p_1}$ with v^1_{EPTS} would have envy of another patient $\mathbf{p_2}$ with v^2_{EPTS} for compatible organ $\mathbf{j}$ with v^j_{KDPI} if $\mathbf{p_2}$ receives the organ. The envy amount is $|v^1_{EPTS} - v^j_{KDPI}|/|v^2_{EPTS} - v^j_{KDPI}|$.
- quality competitiveness: We maximize the number of patients matched to organs of exact quality indices.

Assumptions:

1. for each new organ, there is a patient of exact blood type
2. for each new organ, there is a patient of exact quality index

Property	Assumption	HARDTYPEBESTINDEX	SOFTTYPEBESTINDEX
Blood type efficiency	1	$\checkmark$	$\times$
Quality efficiency	1, 2	$\times$	$\checkmark$
Blood type envy-freeness	1	$\checkmark$	$\times$
Quality envy-freeness	1, 2	$\times$	$\checkmark$
Bound of quality envy-freeness	1	∞	**1**
Blood type competitive ratio	1	**1**	**2**
Quality competitive ratio	1, 2	∞	**1**

Table 2: Overview of properties: $\mathbf{k}$ patients and $\mathbf{l}$ organs.

FOR FURTHER INFORMATION

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